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On the muon anomalous magnetic moment in models with a superlight gravitino*

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ABSTRACT: We perform a general analysis of the contributions to the muon anomalous magnetic moment in models with a superlight gravitino. We discuss the interpretation and the phenomenological implications of the results. We find that present constraints on the model parameters are comparable and complementary to the ones coming from collider searches and from perturbative unitarity. However, the Brookhaven E821 experiment will probe large unexplored regions of parameter space.

KEYWORDS: Supersymmetry Breaking, Supergravity Models, Supersymmetric Effective Theories, Supersymmetric Standard Model.

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Contents

1. Introduction

The anomalous magnetic moment of the muon is both one of the best measured quantities in particle physics and one of the most accurately known predictions of the Standard Model (SM) of strong and electroweak interactions. In terms of $a_\mu \equiv (g_\mu - 2)/2$, the present experimental and theoretical situation can be summarized [1] as follows:

$$\begin{aligned} a_\mu^{ex} &= (116592350 \pm 730) \times 10^{-11}\,, \\ a_\mu^{SM} &= (116591596 \pm \ \ 67) \times 10^{-11}\,. \end{aligned} \tag{1.1}$$

As a consequence, powerful constraints on extensions of the SM can be extracted from the comparison of eq. (1.1) with possible non-standard contributions, δa_μ. The 95% confidence level bound corresponding to the present data is[1]

$$-0.7 \times 10^{-8} < \delta a_\mu < 2.2 \times 10^{-8}\,, \tag{1.2}$$

and the E821 experiment at Brookhaven [2] is expected to reduce further the experimental error by roughly a factor of 20.

[1]More conservative estimates of the present theoretical error associated with the hadronic contribution can enlarge the total theoretical error by up to a factor of two. Given the present experimental precision, this would not affect significantly the bounds of eq. (1.2), and improvements are expected in the future.

The most popular and theoretically motivated extensions of the SM are those incorporating low-energy supersymmetry. The study of a_μ in supersymmetric models has a long history. Already in the early days of supersymmetry it was realized [3] that $a_\mu = 0$ in the limit of unbroken supersymmetry, both in the global and in the local [4] case. In the case of broken supersymmetry, the first estimates of δa_μ from loop diagrams involving supersymmetric particles were given in [5]. By now, the full one-loop contribution to a_μ in the Minimal Supersymmetric extension of the Standard Model (MSSM) is available, for an arbitrary supersymmetric particle spectrum [6, 7]: the relevant one-loop diagrams involve smuon-smuon-neutralino and chargino-chargino-sneutrino loops. Thanks to the fast decoupling properties of the soft supersymmetry-breaking mass terms, these contributions are comfortably within the bounds of eq. (1.2) for typical values of the MSSM parameters allowed by the direct searches for supersymmetric particles. The only situation in which δa_μ can saturate the bounds of eq. (1.2) is when the masses of supersymmetric particles are close to their present lower bounds and $\tan\beta = v_2/v_1$ is very large (so that the muon Yukawa coupling is considerably enhanced).

In supersymmetric models with a light gravitino, the effective low-energy theory does not contain only the MSSM states, but also the gravitino and its superpartners, and we must compute the additional contributions δa_μ coming from the one-loop diagrams where these particles (as well as other MSSM particles) are exchanged on the internal lines. Most of the existing calculations of these effects were performed in the framework of supergravity, and the discussion [8, 9, 10, 11] was mainly focused (with some controversies) on the finiteness of the broken supergravity contributions. The most complete study available so far is the one of ref. [10], which considered a general spontaneously broken supergravity, and computed the one-loop diagrams involving virtual particles from the matter, gauge and gravitational supermultiplets. However, in the case of a light gravitino (the only phenomenologically relevant one for this type of study) we can work directly in the globally supersymmetric limit, keeping only the goldstino and its spin–0 superpartners (sgoldstinos) as the relevant degrees of freedom from the supersymmetry-breaking sector [12]. Indeed, this method has already been partially applied in [6], to compute the contribution to δa_μ of the goldstino-smuon-smuon loops. In this paper, we shall adopt such a method to perform a general analysis of the contributions to δa_μ from the supersymmetry-breaking sector of the theory.

The paper is organized as follows. In sect. 2, we introduce the model and derive the relevant interaction terms. In sect. 3, we present the general expressions for our results. In sect. 4, we discuss their interpretation, with emphasis on possible divergent contributions. In sect. 5, we specialize our formulae to some phenomenologically interesting limits and discuss their implications. Whenever possible, we compare with the previous literature, in particular with the supergravity computation of ref. [10].

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2. The model

We will compute the anomalous magnetic moment of the muon in a general $N = 1$ globally supersymmetric model containing a U 1) gauge vector superfield, $V \equiv (\lambda, A_\mu, D)$, and three chiral superfields, $M \equiv (\tilde{\mu}, \mu, F^M)$, $M^c \equiv (\tilde{\mu^c}, \mu^c, F^{M^c})$, $Z \equiv (z, \psi_z, F^Z)$, with the following charge assignments: $Q(M) = -1$, $Q(M^c) = +1$, $Q(Z) = 0$. Despite its simple field content, the model should reproduce all the relevant aspects of the realistic case. In particular, the U 1) will be associated with the exact gauge symmetry of supersymmetric QED, the M and M^c multiplets will contain the degrees of freedom of the left-handed muon and anti-muon, respectively, and the Z multiplet will contain the goldstino. The most general effective lagrangean with the above field content is determined, up to higher-derivative terms, by a superpotential $w(Z, M, M^c)$, a gauge kinetic function $f(Z, M, M^c)$ and a Kähler potential $K(Z, \overline{Z}, M, \overline{M}, M^c, \overline{M^c})$. We can parametrize such functions as follows:

$$
\begin{aligned}
w &= FZ + \frac{\sigma}{6} Z^3 + mMM^c + \rho ZMM^c + \cdots \,, \\
f &= \frac{1}{e^2}\left(1 + \eta \frac{2\,Z}{\Lambda} + \gamma_f \frac{MM^c}{\Lambda^2} + \cdots\right), \\
K &= |Z|^2 + |M|^2 + |M^c|^2 - \alpha_z \frac{|Z|^4}{4\Lambda^2} - \\
&\quad - \alpha_\mu \frac{|Z|^2|M|^2}{\Lambda^2} - \alpha_{\mu^c} \frac{|Z|^2|M^c|^2}{\Lambda^2} - \gamma_K \frac{\left(\overline{Z}^2 MM^c + \text{h.c.}\right)}{2\Lambda^2} + \cdots \,.
\end{aligned} \tag{2.1}
$$

In the above expressions, all the parameters have been taken to be real for simplicity, and the dots stand for terms that either do not play any rôle in the calculation of a_μ or can be eliminated by (non-linear) analytic field redefinitions.[2] For example, a possible cubic term in K of the form $(\beta_K \overline{Z} MM^c/\Lambda + \text{h.c.})$ can be eliminated by the analytic field redefinition $Z \to (Z - \beta_K MM^c/\Lambda)$, accompanied by the parameter redefinitions $(m - \beta_K F/\Lambda) \to m$ and $(\gamma_f - 2\eta\beta_K) \to \gamma_f$. Other field redefinitions have been used to eliminate other low-order terms and to shift $\langle z \rangle$ to zero. Taking all this into account, the above expressions of w, f and K are the most general ones compatible with a classical vacuum with broken supersymmetry and unbroken CP and U 1). Indeed, the parametrization is still slightly redundant, since one of the dimensionless parameters, e.g. η, can be re-absorbed in the definition of the mass scale Λ characterizing the non-renormalizable operators, but we stick to it to have a clearer physical interpretation of our results.

By standard techniques, it is easy to check that there is a local minimum of the classical potential where supersymmetry is spontaneously broken, with vacuum

[2] The above parametrization corresponds essentially to choosing normal coordinates [13], and simplifies our diagrammatic computations. Although reparametrization covariance is lost in the intermediate steps, general arguments guarantee that the final physical results are the same. We have explicitly checked this, starting from a fully general parametrization.

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energy $\langle V\rangle = F^2$, and the gauge symmetry remains unbroken, with the gauge coupling constant given by $\langle(\mathrm{Re}\, f)^{-1}\rangle = e^2$. Notice that the Kähler metric is canonical at the minimum, so that the component fields of the chiral superfields are automatically normalized. In the fermion sector of the model, ψ_z is the massless goldstino, whereas the muon has a Dirac mass $m_\mu = m$, and the photino has a Majorana mass $M_\lambda = \eta F/\Lambda$. In the scalar sector of the model, the real and imaginary parts of the complex sgoldstino, $z \equiv (S+iP)/\sqrt{2}$, have squared masses:

$$m_S^2 = \alpha_z \frac{F^2}{\Lambda^2} + \sigma F\,, \qquad m_P^2 = \alpha_z \frac{F^2}{\Lambda^2} - \sigma F\,, \tag{2.2}$$

and the smuons have the following squared mass matrix:

$$\mathcal{M}_0^2 = \begin{pmatrix} \tilde{m}_\mu^2 + m_\mu^2 & \delta m^2 \\ \delta m^2 & \tilde{m}_{\mu^c}^2 + m_\mu^2 \end{pmatrix}\,, \tag{2.3}$$

where

$$\tilde{m}_\mu^2 = \alpha_\mu \frac{F^2}{\Lambda^2}\,, \qquad \tilde{m}_{\mu^c}^2 = \alpha_{\mu^c} \frac{F^2}{\Lambda^2}\,, \qquad \delta m^2 = \rho\, F\,. \tag{2.4}$$

We denote the smuon mass eigenstates by $\tilde{\mu}_1 \equiv \cos\theta\tilde{\mu} + \sin\theta\overline{\tilde{\mu}^c}$, $\tilde{\mu}_2 \equiv -\sin\theta\tilde{\mu} + \cos\theta\overline{\tilde{\mu}^c}$, and the corresponding mass eigenvalues by m_1^2 and m_2^2, respectively. We also recall that

$$\sin 2\theta = \frac{2\delta m^2}{m_1^2 - m_2^2}\,. \tag{2.5}$$

In the model defined by eqs. (2.1), the different classes of one-loop Feynman diagrams that may contribute to a_μ are displayed in fig. 1.

The Feynman rules needed to compute all these diagrams are summarized in the following lagrangean

$$\begin{aligned}
\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\overline{\lambda}\overline{\sigma}^\mu\partial_\mu\lambda - \frac{M_\lambda}{2}\left(\lambda\lambda + \overline{\lambda}\,\overline{\lambda}\right) + \frac{1}{2}(\partial^\mu S)(\partial_\mu S) - \frac{1}{2}m_S^2 S^2 + \\
& + \frac{1}{2}(\partial^\mu P)(\partial_\mu P) - \frac{1}{2}m_P^2 P^2 + \overline{(D^\mu\tilde{\mu}_1)}(D_\mu\tilde{\mu}_1) - m_1^2|\tilde{\mu}_1|^2 + \overline{(D^\mu\tilde{\mu}_2)}(D_\mu\tilde{\mu}_2) - \\
& - m_2^2|\tilde{\mu}_2|^2 + i\overline{\psi}_z\overline{\sigma}^\mu\partial_\mu\psi_z + i\overline{\mu}\,\overline{\sigma}^\mu D_\mu\mu + i\overline{\mu^c}\,\overline{\sigma}^\mu D_\mu\mu^c - m_\mu\left(\mu\mu^c + \overline{\mu}\,\overline{\mu^c}\right) - \\
& - \frac{1}{\sqrt{2}}\frac{M_\lambda}{F}\left(i\psi_z\sigma^{\mu\nu}\lambda F_{\mu\nu} + \mathrm{h.c.}\right) - \frac{1}{2\sqrt{2}}\frac{M_\lambda}{F}S\,F^{\mu\nu}F_{\mu\nu} + \frac{1}{4\sqrt{2}}\frac{M_\lambda}{F}P\,\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - \\
& - \frac{1}{\sqrt{2}}\left(\frac{\delta m^2}{F} + \gamma_K\frac{F}{\Lambda^2}\right)\left(S\mu\mu^c + \mathrm{h.c.}\right) - \frac{1}{\sqrt{2}}\left(\frac{\delta m^2}{F} - \gamma_K\frac{F}{\Lambda^2}\right)\left(i\,P\mu\mu^c + \mathrm{h.c.}\right) - \\
& - \frac{1}{F}\left\{\left[(m_1^2 - m_\mu^2)\cos\theta\overline{\tilde{\mu}_1} - (m_2^2 - m_\mu^2)\sin\theta\overline{\tilde{\mu}_2}\right]\mu\psi_z + \mathrm{h.c.}\right\} - \\
& - \frac{1}{F}\left\{\left[(m_1^2 - m_\mu^2)\sin\theta\tilde{\mu}_1 + (m_2^2 - m_\mu^2)\cos\theta\tilde{\mu}_2\right]\mu^c\psi_z + \mathrm{h.c.}\right\} + \\
& + e\sqrt{2}\left[(\cos\theta\overline{\tilde{\mu}_1} - \sin\theta\overline{\tilde{\mu}_2})\mu\lambda - (\sin\theta\tilde{\mu}_1 + \cos\theta\tilde{\mu}_2)\mu^c\lambda + \mathrm{h.c.}\right] - \\
& - \gamma_f\frac{1}{2\sqrt{2}\Lambda^2}\left(i\,\tilde{\mu}\mu^c\sigma^{\mu\nu}\lambda F_{\mu\nu} + i\,\tilde{\mu}^c\mu\sigma^{\mu\nu}\lambda F_{\mu\nu} + \mathrm{h.c.}\right) + \cdots\,,
\end{aligned} \tag{2.6}$$

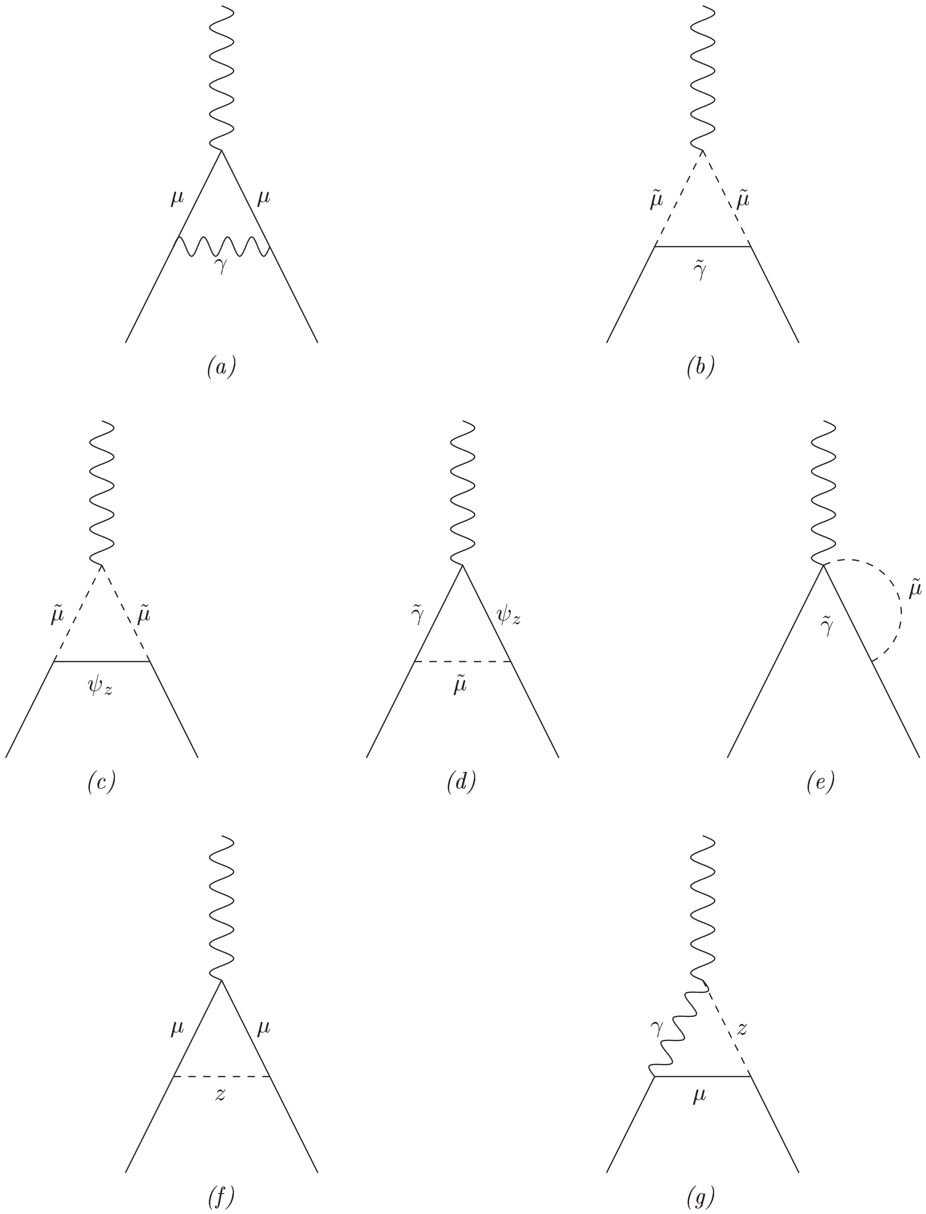

Figure 1: The different classes of one-loop diagrams contributing to a_μ.

where we used two-component spinor notation in the conventions of ref. [14] (with canonically normalized photon and photino fields, and the field redefinition $\lambda \to i\lambda$), and the dots stand for the gauge-fixing term and other terms that are not relevant for the following calculations. In eq. (2.6), $\epsilon^{0123} = -\epsilon_{0123} = -1$, and the covariant derivatives are defined according to the following sign convention: $D_\mu \equiv \partial_\mu + ieQA_\mu$.

To conclude the description of the model, we observe that almost all the parameters controlling the interactions of eq. (2.6) are related with the supersymmetry-breaking scale, $\sqrt{F}$, and the particle spectrum (masses and mixing angles): the only two additional parameters are γ_f and γ_K.

3. General results

We now move to the computation of the different diagrams depicted in fig. 1 and to the presentation of the results, without any assumption on the model parameters.

The diagrams in figs. 1a and 1b are nothing else than the muon-muon-photon and smuon-smuon-photino diagrams of supersymmetric QED, whose contributions are well-known. Fig. 1a gives the one-loop QED result [15]:

$$a_\mu^{(a)} = \frac{\alpha}{2\pi} \, . \tag{3.1}$$

Its supersymmetric counterpart, fig. 1b, gives

$$\begin{aligned}\delta a_\mu^{(b)} &= -\frac{\alpha}{2\pi} m_\mu^2 \left[\int_0^1 dx \frac{x^2(1-x)}{m_1^2 x + M_\lambda^2 (1-x) - m_\mu^2 x(1-x)} + (1 \to 2) \right] + \\ &\quad + \frac{\alpha}{\pi} \frac{m_\mu \, \delta m^2 \, M_\lambda}{m_1^2 - m_2^2} \left[\int_0^1 dx \frac{x(1-x)}{m_1^2 x + M_\lambda^2 (1-x) - m_\mu^2 x(1-x)} - (1 \to 2) \right]\end{aligned} \tag{3.2}$$

Most of the remaining diagrams of fig. 1 involve goldstino or sgoldstino exchanges. Some of them are finite, others lead to potential divergences and require a regularization. We will use two alternative regularizations, appropriate for supersymmetry: either dimensional reduction in D-dimensional momentum space, or a naïve cutoff Λ_{UV} in 4-dimensional momentum space. To keep track of the regularization dependence, we will use the abbreviations:

$$\Delta_{UV} = \begin{cases} \dfrac{2}{4-D} - \gamma_E + \log 4\pi & \text{(dim. reduction)} \\ \log \dfrac{\Lambda_{UV}^2}{\mu^2} - 1 & \text{(mom. cutoff)} \, , \end{cases} \tag{3.3}$$

$$\hat{\Delta} = \begin{cases} 1 & \text{(dim. reduction)} \\ 0 & \text{(mom. cutoff)} \, , \end{cases} \tag{3.4}$$

where γ_E is the Euler-Mascheroni constant and μ is the renormalization scale.

The diagrams of class (c), containing smuon-smuon-goldstino loops, give a finite result:

$$\delta a_\mu^{(c)} = -\frac{m_\mu^2}{16\pi^2 F^2} \left[\int_0^1 dx \frac{(m_1^2 - m_\mu^2)^2 x(1-x)}{m_1^2 - m_\mu^2 (1-x)} + (1 \to 2) \right] . \tag{3.5}$$

The diagrams of class (d), containing photino-smuon-goldstino loops, are superficially divergent. However, the overall result is again finite:

$$\begin{aligned}\delta a_\mu^{(d)} &= -\frac{m_\mu \delta m^2 M_\lambda}{8\pi^2 F^2} \Bigg[\hat{\Delta} + \frac{2 m_\mu^2}{m_1^2 - m_2^2} \int_0^1 dx \int_0^{1-x} dy \times \\ &\qquad \times \left(\frac{(m_1^2 - m_\mu^2) x^2}{m_1^2 x + M_\lambda^2 y - m_\mu^2 x(1-x)} - (1 \to 2) \right) \Bigg] + \\ &\quad + \frac{m_\mu^2 M_\lambda^2}{8\pi^2 F^2} \int_0^1 dx \int_0^{1-x} dy \left(\frac{(m_1^2 - m_\mu^2) x}{m_1^2 x + M_\lambda^2 y - m_\mu^2 x(1-x)} + (1 \to 2) \right) ,\end{aligned} \tag{3.6}$$

where $\hat{\Delta}$ was defined in eq. (3.4). Notice that the first term within brackets is regularization dependent: we will see shortly that it is cancelled by a similar term from another diagram.

The contribution of the photino-smuon diagrams of class (e) is in general divergent:

$$
\begin{aligned}
\delta a_\mu^{(e)} = & -\gamma_f \frac{m_\mu M_\lambda}{16\pi^2\Lambda^2}\Bigg[2\Delta_{UV} - \int_0^1 dx \times \\
& \times \left(\log\frac{m_1^2 x + M_\lambda^2(1-x) - m_\mu^2 x(1-x)}{\mu^2} + (1\to 2)\right)\Bigg] - \\
& -\gamma_f \frac{m_\mu^2 \delta m^2}{8\pi^2\Lambda^2}\frac{1}{m_1^2 - m_2^2}\int_0^1 dx\, x \log\frac{m_1^2 x + M_\lambda^2(1-x) - m_\mu^2 x(1-x)}{m_2^2 x + M_\lambda^2(1-x) - m_\mu^2 x(1-x)}\,,
\end{aligned} \tag{3.7}
$$

where Δ_{UV} was defined in eq. (3.3). If $\gamma_f = 0$, then $\delta a_\mu^{(e)} = 0$.

The diagrams of class (f), containing muon-muon-sgoldstino loops, give a finite result:

$$
\begin{aligned}
\delta a_\mu^{(f)} = & +\frac{m_\mu^2}{16\pi^2}\Bigg[\left(\frac{\delta m^2}{F} + \gamma_K\frac{F}{\Lambda^2}\right)^2 \int_0^1 dx \frac{x^2(2-x)}{m_S^2(1-x) + m_\mu^2 x^2} + \\
& + \left(\frac{\delta m^2}{F} - \gamma_K\frac{F}{\Lambda^2}\right)^2 \int_0^1 dx \frac{-x^3}{m_P^2(1-x) + m_\mu^2 x^2}\Bigg]\,,
\end{aligned} \tag{3.8}
$$

The overall contribution of the diagrams of class (g), containing the photon-muon-sgoldstino loops, is in general divergent:

$$
\begin{aligned}
\delta a_\mu^{(g)} = & \gamma_K \frac{m_\mu M_\lambda}{4\pi^2\Lambda^2}\Bigg[\Delta_{UV} - \frac{1}{2} - \int_0^1 dx \int_0^{1-x} dy \times \\
& \times \left(\log\frac{m_S^2 y + m_\mu^2 x^2}{\mu^2} + (S\to P) + \frac{m_\mu^2 x^2}{m_P^2 y + m_\mu^2 x^2}\right)\Bigg] + \\
& + \frac{m_\mu \delta m^2 M_\lambda}{8\pi^2 F^2}\left[\hat{\Delta} - 2\int_0^1 dx \int_0^{1-x} dy \left(\log\frac{m_S^2 y + m_\mu^2 x^2}{m_P^2 y + m_\mu^2 x^2} - \frac{m_\mu^2 x^2}{m_P^2 y + m_\mu^2 x^2}\right)\right]
\end{aligned} \tag{3.9}
$$

where Δ_{UV} and $\hat{\Delta}$ were defined in eqs. (3.3) and (3.4), respectively. If $\gamma_K = 0$, the result is finite. Notice also that the regularization-dependent contribution in the second line of eq. (3.9) cancels exactly with the one of eq. (3.6).

As a consistency check of our results, we have verified that a_μ vanishes in the limit of exact supersymmetry, as expected on general grounds [3]. We recall that, in the unbroken phase of supersymmetric QED, the overall a_μ is zero, as we can easily check from eq. (3.2) by putting $m_1^2 = m_2^2 = m_\mu^2$ and $M_\lambda = 0$:

$$
\delta a_\mu^{(b)} = -a_\mu^{(a)} = -\frac{\alpha}{2\pi}\,. \tag{3.10}
$$

Similar cancellations take place among the other diagrams. Indeed, if we re-express the spectrum parameters in terms of the original ones and take the appropriate $F \to 0$ limit, we find:

$$\delta a_\mu^{(f)} = -\delta a_\mu^{(c)} = \frac{\rho^2}{16\pi^2}\,, \qquad \delta a_\mu^{(g)} = -\delta a_\mu^{(d)} = \frac{\eta\rho\, m}{8\pi^2\Lambda}(\hat{\Delta}+1)\,, \qquad \delta a_\mu^{(e)} = 0\,. \quad (3.11)$$

Incidentally, this confirms that the cancellation of the terms $\pm m_\mu \delta m^2 M_\lambda \hat{\Delta}/(8\pi^2 F^2)$ in the general results above is just a consequence of supersymmetry.

4. Discussion

We now discuss the interpretation of our results. We start from the observation that, in an effective theory with spontaneously broken global supersymmetry and arbitrary defining functions (w, K, f), the one-loop contribution to a_μ is in general divergent (see eqs. (3.7) and (3.9)). This is in agreement with the results of ref. [10], obtained within the supergravity formalism. It is also consistent with the fact that we could have supplemented our standard (two-derivative) effective supersymmetric lagrangean with higher derivative operators contributing directly to a_μ. An example of such a 'counterterm' is [8]

$$\frac{c}{\Lambda^3}\int d^4\theta \left[\overline{Z} W^\alpha M \overleftrightarrow{D_\alpha} M^c + \text{h.c.}\right], \quad (4.1)$$

where W^α is the supersymmetric field strength, D_α the covariant derivative of supersymmetry and c an arbitrary coefficient. The expansion of (4.1) in component fields contains the operator

$$-\frac{2ic}{\Lambda^3}\overline{F}^{\overline{Z}} \mu\sigma^{\mu\nu}\mu^c F_{\mu\nu} + \text{h.c.}\,, \quad (4.2)$$

which in turn becomes a magnetic moment operator[3] if supersymmetry is spontaneously broken by a non-vanishing VEV for the auxiliary field F^Z.

It is useful to reinterpret the divergent contributions to a_μ found in eqs. (3.7) and (3.9) using a similar language, separating the perturbative generation of supersymmetric operators from the question whether supersymmetry is broken or not. Thus we could say that our effective supersymmetric lagrangean *generates* an operator of the form (4.2) at the one-loop level, with a logarithmically divergent coefficient $\hat{c} = \eta(\gamma_f - 2\gamma_K)\Delta_{UV}/(32\pi^2)$. We can go further and try to characterize such divergences in a more general way, manifestly invariant under analytic field redefinitions. To this purpose, we observe that $\hat{c}$ itself should be viewed as the VEV of a field depen-

[3]For convenience, in this section $F_{\mu\nu}$ denotes the original (not normalized) gauge field. Also, we use the definition $F^Z \equiv Z|_{\theta\theta}$, so that $\langle F^Z \rangle = -F$ in our model.

dent, reparametrization covariant expression, i.e. the operator should be written as[4]

$$-i\frac{\Delta_{UV}}{32\pi^2}\left\langle\frac{\overline{f}_{\overline{Z}}D_M D_{M^c}f}{(\mathrm{Re}f)^2}+2\frac{f_Z R^Z_{\ M\overline{Z}M^c}}{\mathrm{Re}f}\right\rangle\overline{F}^{\overline{Z}}\,\mu\sigma^{\mu\nu}\mu^c F_{\mu\nu}+\mathrm{h.c.}\,, \tag{4.3}$$

where D_i $(i = Z, M, M^c)$ is the scalar field reparametrization covariant derivative and $R^l_{\ i\overline{n}j}$ the curvature tensor of the Kähler manifold:

$$D_iD_jf = f_{ij} - f_l(K^{-1})^{l\overline{m}}K_{\overline{m}ij}\,, \qquad R^l_{\ i\overline{n}j} = (K^{-1})^{l\overline{m}}\left(K_{\overline{m}i\overline{n}j} - K_{\overline{m}\overline{n}p}(K^{-1})^{p\overline{q}}K_{\overline{q}ij}\right). \tag{4.4}$$

For example, with our coordinate choice of eqs. (2.1), only the $\gamma_f MM^c$ term in f and the $\gamma_K\overline{Z}^2MM^c$ term in K contribute to $\langle D_MD_{M^c}f\rangle$ and $\langle R^Z_{\ M\overline{Z}M^c}\rangle$, respectively. We recall that other terms that could contribute to $\langle D_MD_{M^c}f\rangle$ and $\langle R^Z_{\ M\overline{Z}M^c}\rangle$ were removed by suitable field redefinitions. Our general expression (4.3) can be also used to reinterpret the divergent results of [10], where no MM^c term was included in f, but a $\overline{Z}MM^c$ term was present in K. We can view this as an alternative coordinate choice, which leads to a non-vanishing $\langle D_MD_{M^c}f\rangle$ through the connection term [more specifically, from a type-(d) rather than a type-(e) diagram]. We should add that the supergravity calculation of [10] found several divergent contributions in individual diagrams, most of which were cancelling in the final result and two of which were not. In our case, the use of a globally supersymmetric lagrangean (in a convenient field parametrization) avoids the proliferation of divergences and reproduces directly the two 'genuine' ones.

Our covariant formula (4.3) clearly shows the geometrical meaning of the two kind of divergences. We could now ask whether they should be considered as independent or could possibly originate from a single object. In fact, taking into account that $\langle f_M\rangle = \langle f_{M^c}\rangle = 0$, we observe that the VEV in eq. (4.3) is identical to the VEV of $-4D_{\overline{Z}}D_MD_{M^c}\log\mathrm{Re}f$. Thus we are led to conjecture that a (two-derivative) supersymmetric lagrangean like ours, with generic f and K functions, generates radiatively the following logarithmically divergent operator:

$$i\frac{\Delta_{UV}}{8\pi^2}\left(D_{\overline{Z}}D_MD_{M^c}\log\mathrm{Re}f\right)\overline{F}^{\overline{Z}}\mu\sigma^{\mu\nu}\mu^cF_{\mu\nu}+\mathrm{h.c.} \tag{4.5}$$

The corresponding (higher-derivative) superfield operator should be an appropriate generalization of the operator (4.1).

We have discussed so far the structure of the supersymmetric operators that give divergent contributions to a_μ once supersymmetry is broken. Since the existence of such operators limits somewhat our predictive power, we could look for models where

[4]The scalar field dependence of the fermionic terms that we are discussing is similar to that of certain bosonic terms of ref. [16], where the logarithmically divergent bosonic contributions to the one-loop effective action of supergravity models were computed. It is plausible that our terms are supersymmetric partners of some of those of ref. [16], although we were unable to establish such a connection in full detail.

the corresponding coefficients vanish, or are at least suppressed. The simplest case [8, 9, 10] is to have $\langle f_Z \rangle = 0$, implying $M_\lambda = 0$. More interestingly, the whole combination in eq. (4.3) [or the corresponding compact expression in eq. (4.5)] might vanish if f and K were related, either functionally or just on the vacuum (the latter case corresponds to the equality $\gamma_f = 2\gamma_K$, if we use our original parametrization). This approach may lead to models with an underlying extended supersymmetry.

The question could also be addressed in terms of ordinary symmetry arguments. We may ask whether there is a symmetry that allows for non-vanishing values of m_μ and a_μ but forbids the divergent one-loop contributions. A possible candidate is the R-symmetry defined by the following charge assignments: $R(Z) = +2$, $R(MM^c) = +2$ (the separate values of the R-charges of M and M^c are not important here). Such an assignment, however, would put to zero not only the coefficients γ_f and γ_K of the divergent operators (and possible coefficients related to them by analytic field redefinitions compatible with the R-symmetry), but also the mixing terms in the smuon and sgoldstino mass matrices and, most importantly, the photino mass M_λ. With the latter constraint, the construction of a realistic model with the full Standard Model gauge group appears very unlikely. We were unable to find alternative symmetries that forbid the divergent one-loop contributions to a_μ in the presence of a realistic spectrum.

A milder requirement may be to ask for a symmetry that forces the divergent contributions, eqs. (3.7) and (3.9), to be at least proportional to m_μ^2. If so, reasonable choices of the ultraviolet cutoff would produce contributions to a_μ that are at most of the same order of magnitude as the finite ones. An obvious candidate for such a symmetry is a chiral $\mathrm{U}(1)_S$, with the charge assignments: $S(Z) = 0$, $S(MM^c) \neq 0$. Such a symmetry would be explicitly broken by a small parameter ϵ to allow for the muon mass term. As a consequence, also δm^2, γ_f and γ_K would be suppressed by the same small parameter. In particular, we would obtain $\delta m^2 \sim m_\mu A$, with A of order the other supersymmetry-breaking masses. Such a hierarchy would be stable against radiative corrections, as can be easily checked by looking at the one-loop diagrams contributing to m_μ and to δm^2.

In order to discuss in more detail the relative size of the $\mathcal{O}(m_\mu^2)$ contributions to a_μ, we should supplement the chiral power counting with additional information. For example, we could assume[5] that $F/\Lambda^2 \sim \epsilon'$ is small and consider an expansion in both ϵ and ϵ'. Even so, at least two possibilities could be considered for the combined power counting: (i) $\gamma_K \sim \gamma_f \sim \epsilon$, $m_\mu \sim \epsilon\Lambda$; (ii) $\gamma_K \sim \gamma_f \sim \epsilon$, $m_\mu \sim \epsilon\,\epsilon'\Lambda$. In case (i), the contributions to a_μ proportional to γ_K and γ_f (or induced by the counterterm of eq. (4.1)) would be suppressed by a factor ϵ' with respect to the remaining ones, and could be considered as subleading. Then we would obtain an unambiguous result at the leading order. In case (ii), on the other hand, such contributions would be

[5] We thank the referee for raising this point.

formally of the same order of the other ones[6]. Moreover, as we shall discuss in more detail below, phenomenological considerations will lead us to consider, among others, values of F/Λ^2 of order one.

We can summarize the above discussion as follows. The effective supersymmetric low-energy theory can contain higher-derivative terms that contribute to a_μ and do not depend (only) on the spectrum. Such terms could either be present as counterterms since the beginning, or be generated radiatively within the low-energy theory itself, or both. Symmetry considerations could help in keeping the size of such contributions under control, as in the example of a chiral symmetry, possibly combined with an expansion in F/Λ^2. In most phenomenologically relevant situations, however, it seems that only a better knowledge of the underlying microscopic theory could remove the residual ambiguities. Taking into account all this, in the following we will use the finite contributions to a_μ only to derive some 'naturalness' constraints on the model parameters, barring the possibility of accidental (or miraculous) cancellations.

5. Phenomenology

Keeping in mind the discussion of the previous section, we now focus on the class of models in which

$$\langle D_M D_{M^c} f \rangle = 0 \,, \qquad \langle R^Z_{\ M\overline{Z}M^c} \rangle = 0 \,, \tag{5.1}$$

so that the total one-loop contribution to a_μ is finite and determined only by the parameters controlling the spectrum.[7] We specialize the general formulae of sect. 3 to some interesting limits, and we discuss the resulting phenomenological constraints.

We begin with the SQED contribution. For realistic smuon and photino masses, so that the muon mass becomes negligible in the comparison, eq. (3.2) becomes

$$\begin{aligned} \delta a_\mu^{(b)} = & -\frac{\alpha}{2\pi} m_\mu^2 \left[\frac{m_1^6 - 6 m_1^4 M_\lambda^2 + 3 m_1^2 M_\lambda^4 + 2 M_\lambda^6 + 6 m_1^2 M_\lambda^4 \log\left(\frac{m_1^2}{M_\lambda^2}\right)}{6 (m_1^2 - M_\lambda^2)^4} + (1 \to 2) \right] + \\ & + \frac{\alpha}{\pi} \frac{m_\mu \, \delta m^2 \, M_\lambda}{m_1^2 - m_2^2} \left[\frac{m_1^4 - M_\lambda^4 - 2 m_1^2 M_\lambda^2 \log\left(\frac{m_1^2}{M_\lambda^2}\right)}{2 (m_1^2 - M_\lambda^2)^3} - (1 \to 2) \right] . \end{aligned} \tag{5.2}$$

[6]Notice that the doubly suppressed muon mass of case (ii) would naturally arise from a Kähler potential term of the form $\beta_K \overline{Z} M M^c/\Lambda$, with $\beta_K \sim \epsilon$. More generally, case (ii) can be justified by an R-symmetry under which Z, M, M^c are neutral, explicitly broken by the parameter ϵ': then all superpotential parameters would be suppressed by ϵ'. Incidentally, this would also imply the same scaling law for all sgoldstino mass terms, $(m_S^2 \pm m_P^2) \sim F^2/\Lambda^2$.

[7]The finiteness conditions (5.1) generalize those of ref. [10]. In our original parametrization, they amount to set $\gamma_f = \gamma_K = 0$. We will not discuss here the alternative intriguing possibility mentioned in the previous section, corresponding to $\gamma_f = 2\gamma_K \neq 0$. We only recall that also in the latter case the result would be finite and regularization-independent, as long as we use regularizations compatible with supersymmetry. However, it would contain an additional parameter besides those controlling the spectrum.

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Next, we consider the contributions from the smuon-photino-goldstino sector, for $\gamma_f = 0$. Eq. (3.7) gives no contribution, whereas eqs. (3.5) and (3.6) simplify considerably in the phenomenologically relevant limit $m_\mu \ll m_1, m_2, M_\lambda$:

$$\delta a_\mu^{(c)} = -\frac{m_\mu^2 (m_1^2 + m_2^2)}{96\pi^2 F^2}, \tag{5.3}$$

$$\delta a_\mu^{(d)} = -\frac{m_\mu \delta m^2 M_\lambda}{8\pi^2 F^2}\hat{\Delta} + \frac{m_\mu^2}{16\pi^2 F^2}\left[\frac{m_1^2 M_\lambda^2}{m_1^2 - M_\lambda^2}\left(1 - \frac{M_\lambda^2}{m_1^2 - M_\lambda^2}\log\frac{m_1^2}{M_\lambda^2}\right) + (1 \to 2)\right]. \tag{5.4}$$

The result in eq. (5.3) is in full agreement with ref. [6], and disagrees in sign with the supergravity computations of refs. [9, 10]. The result in eq. (5.4) can be compared with a similar expression of ref. [10], where the terms within brackets appear with opposite sign. Also, we recall that the first term of eq. (5.4) is regularization-dependent and disappears when all contributions are summed.

We now discuss the contributions from the muon-photon-sgoldstino sector, eqs. (3.8) and (3.9), for $\gamma_K = 0$. Before taking any kinematical limit, we would like to emphasize that such contributions depend only on the spectrum parameters and the supersymmetry breaking scale, with no more model-dependence than the contributions from the smuon-photino-goldstino sector. This point was apparently overlooked in ref. [10]. However, we have checked that the results for the muon-photon-sgoldstino sector given there agree with ours, once the flat limit is taken and appropriate parameter identifications are made (e.g. $3m_{3/2}(m_\mu - m'') = \delta m^2$). Furthermore, the general results in eqs. (3.8) and (3.9) simplify considerably in the limit of heavy or light sgoldstinos:

$$\begin{aligned}
\delta a_\mu^{(f)} &= +\frac{(\delta m^2)^2}{16\pi^2 F^2} \times \begin{cases} \dfrac{m_\mu^2}{m_S^2}\left(-\dfrac{7}{6} + \log\dfrac{m_S^2}{m_\mu^2}\right) + \dfrac{m_\mu^2}{m_P^2}\left(\dfrac{11}{6} - \log\dfrac{m_P^2}{m_\mu^2}\right) & m_\mu \ll m_P, m_S \\ 1 & m_S, m_P \ll m_\mu \end{cases} \\
\delta a_\mu^{(g)} &= +\frac{m_\mu \delta m^2 M_\lambda}{8\pi^2 F^2}\hat{\Delta} + \frac{m_\mu \delta m^2 M_\lambda}{8\pi^2 F^2} \times \begin{cases} \log\dfrac{m_P^2}{m_S^2} & m_\mu \ll m_P, m_S \\ 1 & m_P, m_S \ll m_\mu \,. \end{cases}
\end{aligned} \tag{5.5}$$

Taking other special limits such as $m_{P(S)} \ll m_\mu \ll m_{S(P)}$ is equally straightforward. We recall that the possibility of superlight sgoldstinos (both of them or just one) has been frequently considered in the superlight gravitino literature [17]. Despite its naturalness problems [18], it may be related with possible dynamical mechanisms for the resolution of the strong-CP and/or the cosmological constant problems.

We conclude this section by confronting the above results with the present and future experimental constraints.[8] In the following, it will be convenient to parametrize

[8] For previous analyses based on the one-loop goldstino contributions of ref. [10], see refs. [11] and [19]. The latter paper studies also the influence of superlight sgoldstinos, but neglects their effects at the one-loop level and only partially includes them at the two-loop level.

the off-diagonal element of the smuon mass matrix as $\delta m^2 \equiv m_\mu A$, as suggested for example by an underlying approximate chiral symmetry (see sect. 4). Thus, although A, as defined by the previous relation, is a free parameter equivalent to δm^2, it is natural to expect that A be not much larger than the other supersymmetry breaking masses. We begin by reconsidering the SQED contribution of eq. (5.2). We recall that such a contribution should be replaced, in a fully realistic model, by the full MSSM contribution [7]: here it will be used only as a benchmark for the new contributions associated with the superlight gravitino and its superpartners. For illustration purposes, we can further simplify eq. (5.2) and consider just a representative case. For example, if the smuons and the photino are roughly degenerate, with a common mass M_S, we obtain

$$\delta a_\mu^{(b)} = -\frac{\alpha}{12\pi}\frac{m_\mu^2}{M_S^2}\left(1+\frac{A}{M_S}\right) \simeq -2\times 10^{-10}\times\frac{(100\,\mathrm{GeV})^2}{M_S^2}\times\left(1+\frac{A}{M_S}\right) . \qquad (5.6)$$

As we can see, sizeable contributions can be obtained only if the sparticle masses are significantly smaller than 100 GeV or if the supersymmetry-breaking mass parameter A is significantly larger than the typical supersymmetry-breaking masses.[9]

To discuss the phenomenological impact of the remaining contributions, it will be useful to parametrize all of them in a uniform fashion

$$\delta a_\mu \equiv \frac{m_\mu^2 M_x^2}{16\pi^2 F^2}\,, \qquad (5.7)$$

where the real parameter M_x^2 has the dimension of a mass squared and can be positive or negative. The above form separates the dependence of δa_μ on the spectrum (through M_x) from that on the supersymmetry-breaking scale $\sqrt{F}$. Incidentally, we recall that the latter scale is related to the gravitino mass $m_{3/2}$, the reduced Planck mass M_P and Newton's constant G_N as follows: $F^2 = 3m_{3/2}^2 M_P^2 = 3m_{3/2}^2/(8\pi G_N)$. The present experimental limit and the future sensitivity on M_x are then shown in fig. 2: the two solid lines correspond to the present upper and lower bounds on δa_μ, see eq. (1.2), whereas the dashed line corresponds to $|\delta a_\mu| = 4\times 10^{-10}$, and gives a measure of the expected future sensitivity.

The information in fig. 2 should be combined with the explicit expression of M_x^2 in terms of the spectrum, which can be easily read off our results. In particular, we can put together our results of eqs. (5.3)–(5.5) and express M_x^2 as the sum of 'goldstino' and 'sgoldstino' contributions,

$$M_x^2 = M_{x(G)}^2 + M_{x(SG)}^2\,, \qquad (5.8)$$

[9]We recall that, in the framework of the MSSM, the role of our effective A parameter would be played by the MSSM parameters A_μ and $\mu\tan\beta$. Thus, for example, enhancement effects could originate from large values of $\tan\beta$ [7].

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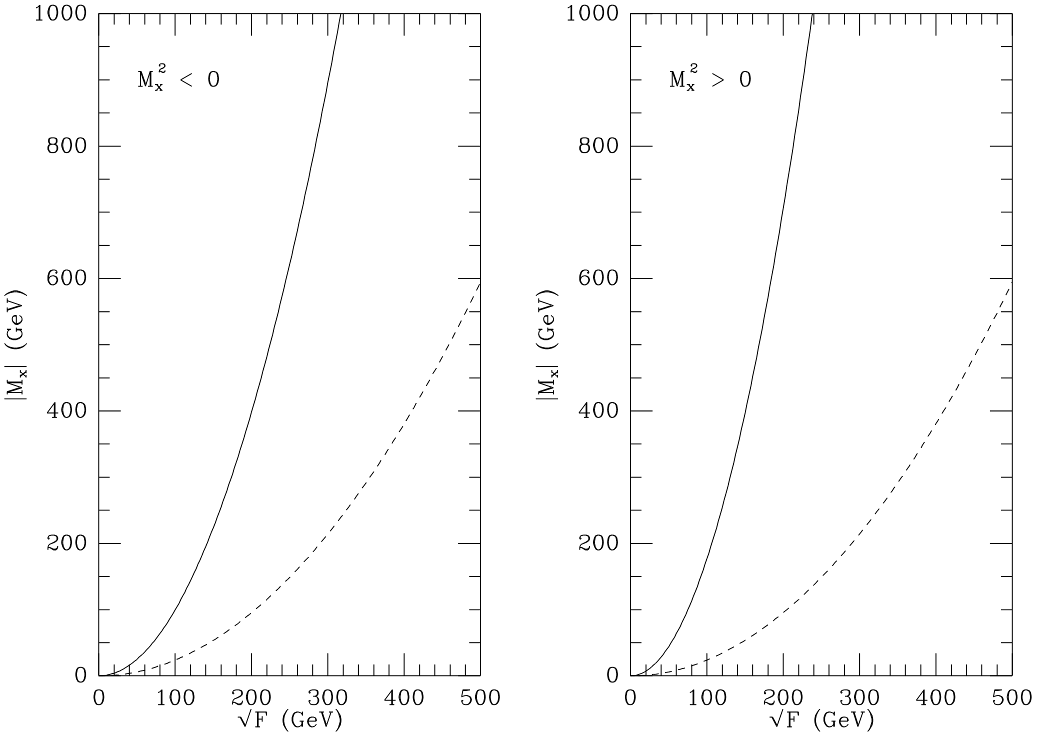

Figure 2: Contours of $\delta a_\mu \equiv m_\mu^2 M_x^2/(16\pi^2 F^2)$ in the plane $(\sqrt{F}, |M_x|)$: the regions above the solid lines correspond to $\delta a_\mu < -70\ (> +220) \times 10^{-10}$, and the dashed lines correspond to $\delta a_\mu = \mp 4 \times 10^{-10}$.

where

$$M_{x(G)}^2 = m_1^2 \left[-\frac{1}{6} + \frac{M_\lambda^2}{m_1^2 - M_\lambda^2} \left(1 - \frac{M_\lambda^2}{m_1^2 - M_\lambda^2} \log \frac{m_1^2}{M_\lambda^2} \right) \right] + (1 \to 2)\,, \tag{5.9}$$

and

$$M_{x(SG)}^2 = \begin{cases} 2AM_\lambda \log \dfrac{m_P^2}{m_S^2} + \mathcal{O}(A^2 m_\mu^2/m_{S,P}^2) & m_\mu \ll m_P, m_S \\ 2AM_\lambda + A^2 & m_S, m_P \ll m_\mu\,. \end{cases} \tag{5.10}$$

Notice that we have already removed from each contribution the regularization-dependent terms that disappear in the overall result.

The size of the goldstino contribution to a_μ depends mainly on the smuon masses, and can be significant only for heavy smuons and a very low supersymmetry-breaking scale. The corresponding sign changes from positive to negative for increasing smuon-to-photino mass ratio. If the smuons and the photino have roughly the same mass M_S, we obtain $M_{x(G)}^2 \simeq 2M_S^2/3$. In interpreting fig. 2, however, we should keep in mind that the situations where $|M_{x(G)}| \gg \sqrt{F}$ correspond to a strong-coupling regime, and are limited by perturbative unitarity. As discussed for example in [14, 18], a reasonable requirement is $|M_{x(G)}| \lesssim (2 \div 3)\sqrt{F}$.

The size of the sgoldstino contribution to a_μ depends on the photino mass, the sgoldstino masses and the parameter A. The latter parameter, which controls the off-diagonal element in the smuon mass matrix, plays a crucial role. In particular, the sgoldstino contribution is zero for vanishing A. Several possibilities can arise for non-vanishing A. We comment here on some representative cases, using the symbol M_S to denote a typical smuon or photino mass. Intermediate cases not discussed below can be easily worked out from our general results.

(i) *Heavy sgoldstinos.* By 'heavy' here we mean the case $m_S, m_P = \mathcal{O}(M_S)$, which is favoured by naturalness considerations [18]. Then from eq. (5.10) we have $M^2_{x(SG)} \simeq 2AM_\lambda \log(m_P^2/m_S^2)$, with either sign being possible. Notice that the sgoldstino contribution becomes negligible if S and P are approximately degenerate. Otherwise, we should also specify the size of A. In the natural case of $A = \mathcal{O}(M_S)$, $M^2_{x(SG)}$ is $\mathcal{O}(M_S^2)$ as well, thus the sgoldstino contribution to a_μ is $\mathcal{O}(m_\mu^2 M_S^2/16\pi^2 F^2)$, like the goldstino one. However, if (for some reason) $A \gg M_S$, the sgoldstino contribution gets enhanced by a factor $\mathcal{O}(A/M_S)$ compared to the goldstino one (for a similar situation in SQED, see eq. (5.6)).

(ii) *Superlight sgoldstinos.* By 'superlight' we mean here the case $m_S, m_P \ll m_\mu$ (however, the qualitative picture remains the same whenever at least one of the sgoldstino masses is $\lesssim m_\mu$). Then from eq. (5.10) we have $M^2_{x(SG)} = 2AM_\lambda + A^2$. Again, if $A = \mathcal{O}(M_S)$, then $M^2_{x(SG)}$ is $\mathcal{O}(M_S^2)$ (either sign being possible), and the sgoldstino contribution is comparable with the goldstino one. If instead $A \gg M_S$, then we have $M^2_{x(SG)} \simeq A^2 \gg M^2_{x(G)}$, and the sgoldstino contribution to a_μ $(= m_\mu^2 A^2/16\pi^2 F^2)$ dominates over the goldstino one. Thus the simultaneous occurrence of superlight sgoldstinos, large A parameter and low supersymmetry-breaking scale gives a potentially large contribution to a_μ, comparable with (or even exceeding) the present experimental bounds. However, it should be taken into account that such a situation is an extreme one, as we have already remarked. Moreover, other potential enhancement effects due to superlight sgoldstinos have been studied in the context of cosmology, astrophysics or collider physics [17], and the inferred lower bounds on $\sqrt{F}$ are already well above the TeV range, weakening the impact of the a_μ constraint.

In summary, since the present lower limits from accelerator searches should be of the order of 100 GeV for the smuon and photino masses,[10] and of the order of 200 GeV for $\sqrt{F}$ [20], we can see that at present a_μ provides a non-negligible but mild

[10] A dedicated analysis for models with a superlight gravitino is not available yet, and would require the generalization of the model to include the full SM gauge group.

indirect constraint. Such a constraint, however, will become much more stringent after the completion of the E821 experiment. Should a discrepancy emerge between the future E821 result and the SM prediction, models with a superlight gravitino may provide a viable explanation.

6. Final comments

We conclude this work with some comments on the predictive power of our effective-theory approach, to clarify its advantages with respect to non-linear realizations of supersymmetry, as well as its residual limitations.

Since a_μ is a low-energy observable involving only SM particles, we may naïvely think that its calculation could be addressed in the context of a 'more effective' theory, where heavy particles have been integrated out and supersymmetry is non-linearly realized. In the case of heavy sgoldstinos, a first step in this direction would be to integrate out only the sgoldstino fields, ending up with a non-linear goldstino superfield [21]. A more radical step would be to integrate out not only the sgoldstinos but also the smuons and the photino, ending up with a fully non-linear realization of supersymmetry [22] on the muon, photon and goldstino fields. It is immediate to realize, however, that non-linear realizations are not a sufficiently predictive approach to address the computation of a_μ. This should be contrasted with other processes involving external goldstinos [14, 20], where the low-energy theorems of spontaneously broken supersymmetry can be put to work. The simplest way to see this point is to look at the dependence of our general finite result (5.7) on the supersymmetry-breaking masses, rewriting it as:

$$\delta a_\mu = \frac{m_\mu^2 M_S^2}{F^2}\, G\left(\frac{m_i^2}{M_S^2}\right) , \tag{6.1}$$

where M_S is a typical supersymmetry-breaking mass that can be taken as a reference scale, the m_i^2 are the individual supersymmetry-breaking mass parameters in the sgoldstino, smuon and photino sectors, and the dimensionless function G encodes detailed information on the supersymmetry-breaking spectrum. From the explicit form of G (see, e.g., eqs. (5.8)–(5.10)), it is evident that there are no decoupling properties for particles with large supersymmetry-breaking masses. This result is expected: decoupling should not occur for heavy particles whose masses break a symmetry that would set the result to zero. In the case of a non-linear realization, we could at most obtain the general scaling law of eq. (6.1), but the function G would be just replaced by a free parameter (if we replace the linear superfield Z by a non-linear goldstino superfield, also the form and the dimension of the lowest-dimensional counterterm for a_μ will change). Starting from an effective theory where some of the superpartners are not present would in general reduce our predictive power, generating new divergences without a prescription on how to regulate them.

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In this respect, removing only the sgoldstinos, while keeping the smuons and the photino, would not lead to any substantial increase in predictive power: sgoldstinos do not have any special decoupling properties with respect to other particles that may acquire large supersymmetry-breaking masses.

The second and last comment is a warning on the correct interpretation of the phenomenological implications of our results. As already discussed in sect. 4, the finite result of our general calculation, obtained by setting $\gamma_K = \gamma_f = 0$, is protected against large corrections only when $\sqrt{F} \ll \Lambda$ and there is a symmetry guaranteeing that $\gamma_K \sim \gamma_f \sim m_\mu/\Lambda$. In this case, the only realistic possibility of generating observable contributions to δa_μ is to have A much larger than the other supersymmetry-breaking masses, to enhance the sgoldstino contributions of eq. (5.10). This is certainly possible, and is the exact counterpart of having large values for A_μ and/or $\mu \tan\beta$ in the MSSM. However, the value of A cannot be pushed too far, since A breaks the same chiral symmetry that protects the smallness of m_μ: too large values for A would make m_μ unnaturally small. Apart from the above possibility, observable contributions to δa_μ can be obtained only when γ_K and γ_f are unsuppressed, in which case we have no predictive power at all, or when $\sqrt{F} \sim \Lambda$. In the latter case, we can still make use of our finite result, but we should keep in mind that we are neglecting other contributions of comparable size. If no deviations from the SM predictions are found, we can still use the results to derive some 'naturalness' constraints on the model parameters, barring the possibility of accidental (or miraculous) cancellations. Should a discrepancy emerge between the experimental data and the SM prediction, however, a detailed quantitative analysis would require a better knowledge of the underlying microscopic theory, to remove the ambiguities associated with the operators controlled by γ_K and γ_f and with possible unsuppressed higher-derivative supersymmetric operators.

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